Number Theory Seminar

The average order of the prime omega function

By

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Abstract:

Write

$$\pi(x) := \sum_{p \leq x} 1$$

for the number of primes up to x. The Prime Number Theorem (PNT) gives us an approximation to this number, precisely it says

$$\pi(x) = \text{li}(x) + \mathcal{O}\left(xe^{-\sqrt{\log x}}\right) \qquad \text{li}(x) := \int_2^x \frac{dt}{\ln(t)}.$$

The proof method was laid out in Riemann's famous memoir and uses the complex analytic properties of the Riemann zeta function $\zeta(s)$. For example, the main term in the above formula comes from the Residue Theorem, picking up a simple residue of $\zeta(s)$ at $s = 1$.

Write $\omega(n)$ for the number of (distinct) prime factors of a number and write

$$W(x) := \sum_{n \leq x} \omega(n).$$

In this talk we discuss the corresponding PNT statement for $W(x)$, leading us to look at $\log \zeta(s)$, whose logarithmic singularity at $s = 1$ means the complex analysis study is more awkward than that of ζ (s).

Date: Tuesday, December 24, 2024
Time: 13:00
Place: SAZ19