Number Theory Seminar

Summation formulas in analytic number theory

By

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Abstract:

Let $\pi(x)$ be the number of primes up to x. If you want to count primes then you can write the count as an integral in the complex plane

$$\pi(x) = \int_{2-i\infty}^{2+i\infty} \frac{\mathcal{P}(s)x^s ds}{s} \qquad \mathcal{P}(s) := \sum_p \frac{1}{p^s}.$$

Analysing the complex analytic properties on the RHS then lets you say something about the arithmetic problem on the LHS - this is the classical proof of the Prime Number Theorem.

The above formula is known as *Perron's formula* and is really an example of a Mellin inversion. In this mini seminar series we'll discuss the formula in this Mellin inversion context and discuss further formulas exemplifying this, especially *Voronoi's summation formula*.

The nature of the talks will be informal and the discussion of Perron and Mellin inversion will be more of an overview rather than details, lasting 2-3 talks. This part should give an idea of what analytic number theory looks like from afar - in particular the hope is it can appeal to undergraduates if they know some complex analysis. In the 2-3 talks after we'll see what it looks like up close by doing Voronoi in more detail.

Date: Friday, Feb 21, 2025
Time: 19:00
Place: SB-Z11