# ODTU-Bilkent Algebraic Geometry

# Distinguishing Brill-Noether loci

**By**

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## Abstract:

Brill-Noether theory has since the end of the 19th century studied linear systems on smooth projective curves (or, equivalently, compact Riemann surfaces). A degree $d$ linear system of dimension $r$ on a curve $C$, called a $g^r_d$, roughly corresponds to a non-degenerate morphism $C \to \mathbb{P}^r$ of degree $d$. In the moduli space $\mathcal{M}_g$ of curves of genus $g$ one can define the *Brill-Noether loci*

$$\mathcal{M}^r_{g,d} := \{C \in \mathcal{M}_g \ : \ C \text{ has a } g^r_d\},$$

which are closed subvarieties.

The classical Brill-Noether-Petri theorem, proved first by D. Gieseker in 1982, states that a general smooth curve $C$ of genus $g$ admits a linear system of dimension $r$ and degree $d$ if and only if the *BrillNoether number*

$$\rho(g,r,d) := g - (r+1)(g-d+r) \geq 0.$$

This can be restated as $\mathcal{M}^r_{g,d} = \mathcal{M}_g$ if and only if $\rho(g,r,d) \geq 0$. When $\rho(g,r,d) < 0$ it is known that the codimension of any component of $\mathcal{M}^r_{g,d}$ is at most $-\rho(g,r,d)$. In general surprisingly little is known about the geometry of BrillNoether loci when $\rho(g,r,d) < 0$, in particular about the containments between them.

I will describe results from a joint work with Asher Auel and Richard Haburcak (arXiv:2406.19993), where we determine all the *maximal* Brill-Noether loci (wrt containment) in terms of numerical conditions on $g, r, d$ and show that they are all distinct.

**Date:** 24 April 2026, Friday
**Time:** 15:40 (GMT+3)
**Place:** Zoom
This is an online seminar. To request the Zoom link, please send a message to sertoz@bilkent.edu.tr