



# TOPOLOGY SEMINAR

## Fusion systems, linking systems and punctured groups

By

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**Abstract:** Saturated fusion systems and associated linking systems are categories modelling the  $p$ -local structure of finite groups. In particular, linking systems contain the algebraic information that is needed to study  $p$ -completed classifying spaces of fusion systems similarly to  $p$ -completed classifying spaces of finite groups. If  $G$  is a finite group and  $S$  is a Sylow  $p$ -subgroup of  $G$ , then we can construct a saturated fusion system  $\mathcal{F}_S(G)$  as follows: The objects are all subgroups of  $S$ , and the morphisms between two objects are the injective group homomorphisms induced by conjugation with elements of  $G$ . Saturated fusion systems which do not arise in this way are called exotic. The concept of a linking system was generalized by Oliver and Ventura to transporter systems. Andrew Chermak introduced moreover group-like structures, called localities, which correspond in a certain way to transporter systems. I will give an introduction to the subject and outline how the theory of localities can be used to prove new theorems on fusion systems. Moreover, I will report on a project with Assaf Libman and Justin Lynd, where we study "punctured groups". Here a transporter system (or a locality) associated to fusion system  $\mathcal{F}$  over  $S$  is called a punctured group if the object set is the collection of all non-identity subgroups. It should be noted in this context that a fusion system  $\mathcal{F}$  over a  $p$ -group  $S$  can be realized as a category  $\mathcal{F}_S(G)$  as above if and only if there is a transporter system whose object set is the full collection of subgroups of  $S$ . In particular, to every group fusion system one can associate a punctured group. In the project with Libman and Lynd, we determine for many of the known exotic fusion systems whether an associated punctured group exists.

**Date:** April 25, 2022

**Time:** 15:00

**Place:** Zoom

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