ODTU-Bilkent Algebraic Geometry

Some Special Torsors and Its Relation to BMY-Inequality

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Abstract:

One of the well-known pathologies in characteristic $p > 0$ is non-smoothness of Picard schemes of surfaces. This pathology is closely related to the failure of Kodaira vanishing theorem (KVT) and of Bogomolov inequality; it was conjectured that a modified form of the Bogomolov-Miyaoka-Yau inequality which contains a correction term measuring non-smoothness of the Picard scheme holds for all surfaces in characteristic p ([4]). In the light of the main result in ([1]), it appears that this correction term, if exists, should in fact account for the non-ordinarity of the Picard scheme.

It was proved by Mukai ([2], Proposition 1.1, [3], Thm.2)) that the failure of KVT on a smooth variety X of arbitrary dimension, is equivalent to the existence of a certain purely inseparable finite cover of X of degree p. In a related setting, in the work on the failure of Bogomolov's inequality $c_1^2(E) \leq 4c_2(E)$ for a semi-stable rank two vector bundle E on a surface X, one of the key ingredients is the construction of a purely inseparable cover of X ([5]). This fact provides the motivation for this talk; we address the existence of a special type of inseparable Galois covers, namely the principal homogeneous spaces on X for groups of type $\mathcal{G}_{a,b}$.

These covers contain as a particular case the α_L-covers of X. We prove the triviality of such covers for suitable pairs (X, L) of a surface X and a line bundle L on X. We will conjecture that if X is an ordinary surface, then for any semi-stable vector bundle E on the surface X we have $c_1^2(E) \leq 4c_2(E)$ by combining the works of Mukai ([2]) and Shepherd-Barron ([5]).

References

[1] J.Jang, *Generically ordinary fibrations and a counterexample to Parshin's conjecture*, Mich.Math.J. 59 (2010), 169-178.

[2] S.Mukai, *Counterexamples of Kodaira's vanishing and Yau's inequality in positive characteristic*, RIMS Preprint no. 1736, Kyotro University, Dec.2011.

[3] S.Mukai, *Counterexamples to Kodaira's vanishing and Yau's inequality in positive characteristics*, Kyoto Journal of Mathematics, Vol. 53, No. 2 (2013), 515–532.

[4] A.N.Parshin, *Letter to Don Zagier by A.N.Parshin*, in Arithmetic Algebraic Geometry, PM 89 (1991), Birkhauser-Verlag, 285-292.

[5] N.I.Shepherd-Barron, *Unstable vctor bundles and linear systems on surfaces in characteristic p*, Inv.Math. 106 (1991), 243-262.

Date: 19 November 2021, Friday
Time: 15:40 (GMT+3)
Place: Zoom

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