ODTU-Bilkent Algebraic Geometry

800 conics in a smooth quartic surface

By

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Abstract: In Generalizing Bauer, define N_{2n}(d) as the maximal number of smooth rational curves of degree d that can lie in a smooth degree-2n K3-surface in P^{n+1}. (All varieties are over C.) The bounds N_{2n}(1) have a long history and currently are well known, whereas for d=2 the only known value is N_6(2)=285 (my recent result reported in this seminar). In the most classical case 2n=4 (spatial quartics), the best known examples have 352 or 432 conics (Barth and Bauer), whereas the best known upper bound is 5016 (Bauer with a reference to Strømme).

For d=1, the extremal configurations (for various values of n) tend to exhibit similar behavior. Hence, contemplating the findings concerning sextic surfaces, one may speculate that -- it is easier to count *all* conics, both irreducible and reducible, but -- nevertheless, in extremal configurations all conics are irreducible. On the other hand, famous Schur's quartic (the one on which the maximum N_4(1) is attained) has 720 conics (mostly reducible), suggesting that 432 should be far from the maximum N_4(2). Therefore, in this talk I suggest a very simple (although also implicit) construction of a smooth quartic with 800 irreducible conics.

The quartic found is Kummer in the sense of Barth and Bauer: it contains 16 disjoint conics. I conjecture that N_4(2)=800 and, moreover, 800 is the sharp upper bound on the total number of conics (irreducible or reducible) in a smooth spatial quartic.

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Time: 15:40
Place: Zoom

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