Bilkent University

Department of Mathematics

ANALYSIS SEMINAR

Fall 2019

Gökhan Yıldırım
Bilkent University

A one-dimensional probabilistic packing problem

Abstract: Consider n molecules lined up in a row. From among the $n-k+1$ nearest neighbor k-tuples, we select one uniformly randomly and bond the k molecules together. Then from the remaining nearest neighbor k-tuples, we select one uniformly randomly and bond the k molecules together. We continue this way until there are no nearest-neighbor k-tuples left.
Let the random variable $M_{n;k}$ count the number of bonded molecules, and let $\mathbf{E}(M_{n;k})$ denote the the expected value of $M_{n;k}$.
I will present the proof of the following result [1]:

Theorem. *(R. G. Pinsky) For each integer $k \geq 2$,*

$$\lim_{n\to\infty} \frac{\mathbf{E}(M_{n;k})}{n} = ke^{-2\sum_{j=1}^{k-1}\frac{1}{j}} \int_0^1 e^{2\sum_{j=1}^{k-1}\frac{s^j}{j}}\, ds.$$

Furthermore, $\frac{M_{n;k}}{n}$ satisfies the weak law of large numbers.

The result for $k = 2$ goes back to an article in 1939 by Paul Flory, 1974 Nobel Laureate in Chemistry.
Some open problems will be discussed at the end of the talk.

[1] R. G. Pinsky. Problems from the Discrete to the Continuous-Probability, Number Theory, Graph Theory, and Combinatorics, Springer.

Date: Monday, October 7, 2019

Time: 14:00-15:00

Place: SA-Z18

Tea and cookies will be served AFTER the seminar. All are most cordially invited.