TOPOLOGY SEMINAR

Mod-ℓ homotopy type of the classifying space for commutativity

By

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Abstract: The classifying space for commutativity, denoted by $B_\text{com} G$, of a Lie group G is assembled from commuting tuples in G as a subspace of the usual classifying space BG. The resulting space classifies principal G-bundles whose transition functions generate an abelian subgroup of G whenever they are simultaneously defined. The relationship between the homotopy type of G and the space $B_\text{com} G$ is much more interesting, and non-trivial compared to the case of BG. In this talk, I will present a work, joint with Ben Williams, where we study the mod-ℓ homotopy type of B_\text{com} G$ at a prime ℓ. The techniques involve a homotopy colimit decomposition over a topological category generalizing the construction of Adem-Gomez and application of results on mapping spaces between classifying spaces of compact Lie groups due to Dwyer-Wilkerson. We show that for a connected compact Lie group the mod-ℓ homotopy type of $B_\text{com}G$ depends on the mod-ℓ homotopy type of BG. More precisely, a mod-ℓ homology isomorphism between the classifying spaces of two such groups induces a mod-ℓ homology isomorphism between the corresponding classifying spaces for commutativity.
Moreover, a Mislin-Friedlander type equivalence, replacing the Lie group by the discrete group obtained from the associated Chevalley group, fails in general, but can be restored after passing to the homotopy orbits by an action of G.

Date: May 22, 2019 Wednesday
Time: 13:40 – 14:40
Place: SA141 Mathematics Seminar Room

* Tea and cookies will be served after the talk. All are most cordially invited.