Syllabus for the Bilkent University Mathematics PhD Programme Qualifying Exam

Latest revision: 13 February 2026.

Written Exam: There are four subject areas:

Section A: Analysis.

Section B: Algebra.

Section C: Applied Mathematics

Section D: Topology and Geometry.

At least 6 weeks before taking the Written Exam, the supervisor of the candidate must select 2 sections. From each of those 2 sections, there are 6 questions and the candidate is to submit up to 4 solutions to those 6. Thus, the candidate is submit up to 8 solutions in total.

As guides for preparation, the main syllabi topics for the four subject areas can be found on the following pages.

Oral Exam: The candidate is to make a presentation directed as if towards a general audience. This is to be followed by questions from the Jury. The questions need not be confined to the topic of the presentation. Some of the questions may be requests for pedagogical clarification of basic concepts. The assessment will be for teaching and communication skills as well as competence in the area of study. Credit will be awarded for ability to give simple direct answers to straightforward questions. Scant credit will be awarded for eloquent recitation of profound strategy.

Section A: Analysis

Real Analysis:

1. Algebras of sets. Borel sets.

2. Measures and outer measures. Measurable and nonmeasurable sets.

3. Measurable functions. Lusin and Egorov theorems. Modes of convergence.

4. Integration. Limit theorems for Lebesgue integral. Spaces of integrable functions and their duals.

5. Signed and complex measures. Radon-Nikodym theorem. Hahn, Jordan and Lebesgue decompositions.

6. Differentiation of Lebesgue integral. Functions of bounded variation. Absolutely continuous functions.

7. Product measures. Tonelli-Fubini theorems.

8. Hilbert spaces. Orthogonality. Projections. General Fourier series.

9. Banach spaces. Linear functionals and linear operators. Hahn-Banach theorem. Baire Theorem. Banach-Steinhaus theorem. Open mapping theorem.

Main reference: W. Rudin, "Real and Complex Analysis", McGraw-Hill Int.Editions, 3rd edition, 1987, Chapters 1 to 8.

Supporting reference: Piermarco Cannarsa, Teresa D'Aprile, "Introduction to Measure Theory and Functional Analysis", Springer, 2015.

Complex Analysis:

A. Holomorphic Functions: Complex derivatives, Cauchy-Riemann equations, Cauchy theorem and integral formula, Morera theorem, power series representation, sequences of holomorphic functions, uniqueness theorem, open mapping theorem, maximum modulus principle, winding numbers, Cauchy theorem for multiply connected regions, Cauchy-type integrals.

B. Singularities: Isolated singularities, Laurent series, residue theorem, argument principle, Rouché theorem, residues at infinity, evaluation of integrals and sums.

C. Conformal Mapping: Preservation of angles, Schwarz-Pick lemma, mapping by Möbius transformations, normal families, Riemann mapping theorem, simply connected regions.

D. Harmonic Functions: Maximum principle, mean value property, Poisson integral, Harnack theorem.

E. Infinite products: Convergence, Weierstrass factorication theorem, Jensen formula, Blaschke products.

Main reference: David C. Ullrich, "Complex Made Simple", AMS, 2008, Chapters 0–11, 13.

Supporting references:

J. B. Conway, "Functions of One Complex Variable", 2nd ed., Springer, 1978.
T. W. Gamelin, "Complex Analysis", Springer, 2001.
R. E. Greene, S. G. Krantz, "Function Theory of One Complex Variable", 3rd ed., AMS, 2006.
W. Rudin, "Real and Complex Analysis", 3rd ed., McGraw-Hill, 1987.

Section B: Algebra

Group Theory: Permutation sets, Sylow's Theorem, Jordan–Hölder Theorem. Simplicity of alternating groups. Semidirect product. Orthogonal and unitary groups. Familiarity with some groups of small order. Finitely generated abelian groups.

Galois Theory: Polynomial rings over fields. Eisenstein's Criterion, Artin's Primitive Element Theorem, Fundamental Theorem of Galois Theory, Unsolvability of the Quintic. Cyclotomic polynomials. Calculation of Galois groups for specific polynomials.

Ring Theory: Finitely generated modules over a principal ideal domain. Semisimple, Artinian and Noetherian rings and modules. Jacobson radical. Artin–Wedderburn Structure Theorem. Applications of Zorn's Lemma.

Main references:
- D. S. Dummit, R. M. Foote, "Abstract Algebra", 3rd edition, (John Wiley, 2004).
- T. Y. Lam, "A First Course in Non-Commutative Rings", (Springer, Berlin, 1991).

Supportive references:
- I. Martin Isaacs, "Algebra, a Graduate Course", (American Mathematical Society, providence, 1994).
- Joseph Rotman, "Galois Theory", 2nd edition, (Springer, New York, 1998).

Section C: Applied Mathematics

Linear and Metric Spaces. Normed vector spaces, inner product spaces. Spectral theory. Least-squares solutions and pseudo inverses, singular value decomposition. Metric spaces and completeness. Function spaces, Banach and Hilbert spaces. Orthogonal polynomials, generalized Fourier series.

Linear Operators. Bounded linear operators, adjoint operators, the Fredholm alternative. Compact operators and their spectral theory.

Integral Equations. Integral operators. Hilbert-Schmidt operators. Resolvent and pseudo-resolvent kernels.

Differential Operators. Green's functions. Sturm-Liouville theory. Theory of distributions. Weak convergence. Distributional derivatives and weak solutions of differential equations. Eigenfunction expansions and least-squares solutions.

Calculus of Variations. Fréchet derivative. Euler-Lagrange equations. Constrained problems. Hamilton's principle.

Transform and Spectral Theory. Spectrum of an operator. Transform pairs. Fourier and Laplace transforms. Z-transform.

Partial Differential Equations. Poisson's equation, Laplace equation. Wave and heat equations. Fundamental solutions. Transform methods. Separation of variables.

Approximation Methods. Asymptotic expansions. Regular and singular perturbation theory. The method of averaging. Secular terms and the Poincaré-Lindstedt method.

Stability and Bifurcations. Systems of differential equations. Stability of solutions. Local and global stability. Saddle-node and Hopf bifurcations.

Main reference:
• *Principles of Applied Mathematics* by James P. Keener (CRC Press–Taylor & Francis, 1988). (This book can be freely downloaded from Bilkent campus via this link). There is also a revised 2000 version, also freely accessible from Bilkent campus at this link, which, however, has two pages missing (pp. 140–141). The missing pages can be separately downloaded from the author's homepage).
Reference for the "Stability and Bifurcations" part:
• Chapters 6–8 of *Differential Equations, Dynamical Systems & An Introduction to Chaos* by Morris W. Hirsch, Stephen Smale, and Robert L. Devaney, Elsevier Press.

Section D: Topology and Geometry

Algebraic Topology:

Fundamental group, Van Kampen's Theorem, Covering spaces. Homotopy groups. Singular homology: Homotopy invariance, homology long exact sequence, Mayer–Vietoris sequence, excision. Cellular homology. Homology with coefficients. Simplicial homology and the equivalence of simplicial and singular homology. Axioms of homology. Homology and fundamental group. Simplicial approximation. Cohomology of spaces, Universal Coefficient Theorem, Cup product, Künneth formula. Topological Manifolds.

Main reference: A. Hatcher, *Algebraic Topology* (2000).

Supportive references:
J. R. Munkres, *A First Course in Topology* (Chapter 8).
J. R. Munkres, *Elements of Algebraic Topology.*
G. Bredon, *Geometry and Topology.*
J. J. Rotman, *An Introduction to Algebraic Topology.*
E. Spanier, *Algebraic Topology.*
M. Greenberg and J. Harper. *Algebraic Topology.*
W. S. Massey, *A Basic Course in Algebraic Topology.*

Algebraic Geometry:

Affine varieties, projective varieties, morphisms, rational maps, blow-ups, resolution of simplest singularities, nonsingular varieties, nonsingular curves, intersections in projective space. [Hartshorne, chapter I]

The Riemann–Hurwitz formula, The Riemann–Roch theorem for compact complex Riemann surfaces and its application to low genus curves, Abel's theorem and its applications. [Griffiths, chapters III, IV, V]

Main reference: Hartshorne, Algebraic Geometry, Springer-Verlag, 1977.

Supportive reference: Griffiths, Introduction to Algebraic Curves, AMS, 1989.