Number Theory Seminar

On the sum of the k-th powers of positive integers that are coprime to n

By

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Abstract:

For given positive integers n and k, the sum of the k-th powers of the first n consecutive integers can be given as

$$S_k(n) = 1^k + \cdots + n^k = \sum_{m=1}^{n} m^k.$$

Similarly, we define the sum of the k-th powers of the first $\varphi(n)$ positive integers that are coprime to n as

$$\varphi_k(n) = \sum_{\substack{a=1 \\ (a,n)=1}}^{n} a^k,$$

where $\varphi(n)$ denotes the Euler-phi function. It is well-known that for all $n > 0$ the sum $S_1(n)$ divides $S_k(n)$ when k is odd. Motivated by this result, in this talk we deal with the positive integer values of k for which the sum $\varphi_1(n)$ divides $\varphi_k(n)$ for all $n > 0$. More generally, we define the set

$$\mathcal{D}_s = \{s \leq k : \varphi_s(n) \mid \varphi_k(n) \quad \forall n \geq 1\}.$$

Using certain smooth numbers in short intervals together with the properties of Bernoulli numbers, we show that for any given positive integer s, the set $\mathcal{D}_s$ is finite. Apart from that, we mention several properties related to the set $\mathcal{D}_1$. In particular, we find that the set $\mathcal{D}_1$ contains $\{1, 3, 15\}$. With the aid of a computer algebra toolbox, we conclude that $\mathcal{D}_1 \cap [1, 5000] = \{1, 3, 15\}$.

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