Bilkent University
Department of Mathematics

Summer Projects

2024

Course Committee: A. Gheondea, Ö. Ünlü, G. Yıldırım

Course Coordinator: G. Yıldırım

Project Presentations

August 5, Monday

13:00–13:30: Ata Cankut Akyüz - (Yılmaz)

13:30–14:00: Muhammed Emin Ovat - (Yılmaz)

14:00–14:30: Hena Üçkum - (Ünlü)

14:30–14:45: Break

14:45–15:15: Ekrem Ulusan - (Atay)

15:15–15:45: Muzaffer Öz - (Özsarı)

15:45–16:15: Ziya Kağan Akman - (Gheondea)

August 6, Tuesday

13:00–13:30: Emir Yusuf Akan - (Saldı)

13:30–14:00: Kazım Tuğşad Koyuncu - (Saldı)

14:00–14:30: Umut Selim Seyithanoğlu - (Yıldırım)

14:30–14:45: Break

14:45–15:15: Asım Ocak - (Güloğlu)

15:15–15:45: Zeynep Su Koç - (Heydarzade)

15:45–16:15: Arda Erkan - (Heydarzade)

Primitive Idempotents of Burnside Ring

Ata Cankut Akyüz

Supervisor: Deniz Yılmaz

Oral presentation jury members: Laurence Barker and Çisil Karagüzel

The Burnside ring $B(G)$ of a finite group G is defined as the Grothendieck group of the category of finite G-sets. In this project, our aim is to study the formula for the primitive idempotents of $\mathbb{Q}B(G) = \mathbb{Q}\otimes_{\mathbb{Z}} B(G)$. We first show that primitive idempotents of $\mathbb{Q}B(G)$ are indexed by the conjugacy classes of subgroups of G. Then, we provide a formula for primitive idempotents in terms of transitive G-sets.

The Burnside Ring as a Biset Functor

Muhammed Emin Ovat

Supervisor: Deniz Yılmaz

Oral presentation jury members: Laurence Barker and Çisil Karagüzel

Biset functors, introduced by Bouc, are central to the heart of functorial representation theory of finite groups. This project introduces the Burnside ring as a biset functor.

We begin by exploring G-sets and (H, G)-bisets, including their key features and notions. Then, we define the Burnside group and Burnside ring. Next, we present the necessary preliminaries from category theory, leading to the definition of the biset category and biset functors. Our main aim is to demonstrate that the map assigning to each finite group G its Burnside ring is a biset functor. Additionally, we provide examples of biset functors, including the Burnside functor itself, to demonstrate these concepts in practice. Finally, we define simple functors to understand their algebraic structure and discuss them briefly without exploring the topic in depth.

Uniqueness of the Right Adjoint Functor

Hena Üçkum

Supervisor: Özgün Ünlü

Oral presentation jury members: Laurence Barker and Çisil Karagüzel

In category theory, adjoint functors between two categories are a pair functors with strong structure preserving properties. A pair F, G is denoted as $F \dashv G$. In this project, the uniqueness of adjoint functors will be shown. Categories, functors, natural transformations, Yoneda Lemma and various approaches to adjoints will be introduced. At the end, these will enable us to prove that if $F \dashv G$ and $F \dashv G'$, then G, G' should be unique up to isomorphism.

Modeling the Dynamics of Human Balancing Using Delay Differential Equations

Ekrem Ulusan

Supervisor: Fatihcan Atay

Oral presentation jury members: Kadri İlker Berktav and Naci Saldı

This project presents a comprehensive study on the dynamics of human balancing, modeled using delay differential equations (DDEs). The project covers the rationale behind using DDEs for this purpose, the theoretical background of DDEs, methods for solving and analyzing them, and their application to human balancing dynamics. It is aimed to construct new approaches for the modeling via analyzing previously used methods and understanding the manner of the human balancing system. Finally, this research covers developing a model for the human balancing system and analyzing the stability conditions of the developed model with a control theoretic approach.

Distribution Theory

Muzaffer Öz

Supervisor: Türker Özsarı

Oral presentation jury members: Aurelian Gheondea and Naci Saldı

Distributions, introduced formally by the Soviet mathematician Sergei Sobolev, play a vital role in finding weak solutions to partial differential equations. This project introduces distributions and Fourier transformations in distributions. We initiate the project by reviewing the Lebesgue measure as quickly as possible to obtain the necessary results. Then, we introduce the space of testing functions, defining distribution space and its properties. Next, we present the definition of the Schwartz space to define the tempered distributions. We improve the connection between those defined notions as we move on. Finally, we introduce the Fourier transform in the Schwartz space and, consequently, in the space of tempered distributions.

Self-Similar Measures and Hausdorff Measure (math 391)

Ziya Kağan Akman

Supervisor: Aurelian Gheondea

Oral presentation jury members: Naci Saldı and Gökhan Yıldırım

The aim of this project is to understand a special class of measures called self-similar measures on self-similar structures. First we show a general construction of self-similar measures. From here, we define Bernoulli measure on the shift space. Then we generalize it by defining self-similar measures on arbitrary self-similar structures. Later, we define the abstract version of Hausdorff measure by using Carathéodory's construction and we prove some its measure-theoretical properties. Finally, we construct Hausdorff measure on Euclidean spaces and we define the Hausdorff dimension of an arbitrary set in $\mathbb{R}^d$.

Introduction to Bandit Algorithms for Stochastic Bandits with Finitely Many Arms

Emir Yusuf Akan

Supervisor: Naci Saldı

Oral presentation jury members: Özgün Ünlü and Gökhan Yıldırım

Stochastic bandit problems belong to the type of problems of decision-making with uncertainty. Particularly, it models exploration-vs-exploitation problems which is under the theory of reinforcement learning. Main goal of this project is to understand the Upper Confidence Bound algorithm. This project introduces key definitions and results necessary for the theory of stochastic bandits with finitely many arms such as the regret, the decomposition of regret, environment classes, and subgaussian random variables. The Cramér-Chernoff method is given for concentration analysis of sum of random variables. Using the tools derived, the Explore-then-Commit algorithm and the Upper Confidence Bound algorithm are intuitively constructed and their regret analyses are given. Ultimately, their performances are compared and further theory is outlined.

An Algorithm For Computing Nash Equilibrium of Two-Person

Kazım Tuğşad Koyuncu

Supervisor: Naci Saldı

Oral presentation jury members: Özgün Ünlü and Gökhan Yıldırım

A two-person zero-sum game can be represented by its payoff matrix, where Player 1 selects one of the m rows and Player 2 selects one of the n columns. If Player 1 chooses the i^{th} row and Player 2 chooses the j^{th} column, Player 1 receives $-a_{ij}$ and Player 2 receives a_{ij}. In this project, we will begin by proving the minimax theorem, which demonstrates the existence of an equilibrium solution for matrix games. After establishing the minimax theorem, we will explore an iterative method that converges to the equilibrium solution. This method is crucial for practical applications, as it provides a systematic way for players to reach equilibrium through repeated play.

A Central Limit Theorem for Set Partitions

Umut Selim Seyithanoğlu

Supervisor: Gökhan Yıldırım

Oral presentation jury members: Alexander Degtyarev and Naci Saldı

In combinatorial mathematics, set partitions are a central topic due to their rich structural properties and diverse applications, including in statistical mechanics and random graph theory. This project focuses on a specific aspect of set partitions: the distribution of the number of blocks. The primary goal is to establish a central limit theorem (CLT) for the number of blocks in set partitions.

The proof relies on an elegant and powerful result: if the probability generating function (PGF) of a random variable has only real roots, then the distribution of that variable is asymptotically normal. L. H. Harper(1967) applied this criterion to the PGF associated with the number of blocks in set partitions, demonstrating that the roots of this generating function are real. This result implies the asymptotic normality of the block count. The approach showcases a blend of combinatorial analysis and probabilistic techniques, highlighting the interplay between these fields.

Additionally, we explore Stam's algorithm, a well-known method for generating set partitions uniformly at random.

Primes of the form $x^2 + ny^2$

Asım Ocak

Supervisor: Ahmet Güloğlu

Oral presentation jury members: Tomos Parry and Hamza Yeşilyurt

In this project, we try to understand the historical developments and attempts to solve a problem, namely the primes of the form $x^2 + ny^2$. Specifically, we are researching the work between Fermat and Gauss. In the research, we derive the formulas and theorems they have used to attack the problem. That is, we demonstrate the mathematical tools developing over the course of history of the problem.

Wormhole Geometries in Einstein's Theory of General Relativity

Zeynep Su Koç

Supervisor: Yaghoub Heydarzade

Oral presentation jury members: Metin Gürses and Bülent Ünal

Starting with the Einstein field equations and imposing the requirements of a traversable wormhole, a class of wormholes is presented. For these wormhole solutions, it is required that the wormholes are asymptotically flat and do not possess horizons anywhere. The latter constraint results in a huge radial pressure τ_0 near the wormhole throat. Moreover, this radial pressure and the mass-energy density ρ_0 must satisfy the relation $\tau_0 > \rho_0 c^2$. This relation means a violation of the "energy conditions" at a classical level, particularly the weak energy condition. However, there are already some quantum mechanical phenomena that violate these conditions.

Linearised General Relativity and Gravitational Waves

Arda Erkan

Supervisor: Yaghoub Heydarzade

Oral presentation jury members: Metin Gürses and Bülent Ünal

This project aims to understand the physical meaning and effects of gravitational waves. In order to understand the gravitational waves, the linearisation process must be investigated in depth. To that aim, after mentioning some prior knowledge of general relativity and differential geometry, the linearised Einstein field equations (EFE) are derived, and solutions for the vacuum and the general case are shown. Then, some general concepts in classical field theory are applied to the general linearised EFE solution to obtain the compact-source approximation, stationary, and static source solutions. Lastly, the physical meaning of these gravitational waves is given by showing their effects on free particles and understanding how they are generated or how energy flows in them.