Bilkent University
Department of Mathematics

Summer Projects

2023

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Course Coordinator: G. Yıldırım

Summer Project Presentations

August 15, Tuesday over Zoom

09:30–10:00: Kadir İhsan Sayıcı-(Yılmaz)

10:00–10:30: Ziya Kağan Akman-(Ünlü)

10:30–11:00: Behzat Deniz Özyörük-(Ünlü)

11:00–11:30: Adem Eren Uyanık-(Gheondea)

11:30–12:00: Alper Akyüz-(Gheondea)

12:00–13:00: Lunch Break

13:00–13:30: Bahar İdil Ongun-(Yıldırım)

13:30–14:00: Faruk Beygo-(Yıldırım)

14:00–14:30: Alihan Serim-(Saldı)

14:30–15:00: Serkan Doruk Hazinedar-(Heydarzade)

1 Project Abstracts

1 Kadir İhsan Sayıcı (Supervisor: Deniz Yılmaz)

Classification of saturated fusion systems on cyclic and Klein four groups.

Abstract: The notion of saturated fusion systems is an important concept in the representation theory of finite groups and homotopy theory. Loosely speaking, a (saturated) fusion system on a p-group P, where p is a prime, is a category whose objects are subgroups of P and whose morphisms are injective group homomorphisms and are based on conjugation with satisfying certain axioms. In this talk, we first introduce saturated fusions systems and give an important class of examples arising from Sylow p-subgroups of finite groups. We then study a well-known theorem and its proof which we present these studies in order to describe all saturated fusion systems on an abelian p-group. As applications of this theorem, we study the classification of saturated fusion systems on Klein four group and some cyclic p-groups.

2- Ziya Kağan Akman (Supervisor: Özgün Ünlü)

Pontryagin Construction

Abstract: In this project, the focus will be on Pontryagin construction while introducing the basics of the theory of smooth manifolds embedded in real Euclidean spaces, framed cobordism, and homotopy theory. The main objective is to understand the proof of Pontryagin's work, which states that the framed cobordism classes of n-dimensional manifolds, which have framings on their normal bundles, in a closed manifold M are in a bijective correspondence with the homotopy classes of maps from M to S^n.

3- Behzat Deniz Özyörük (Supervisor: Özgün Ünlü)

Density Theorem

Abstract: In this project, we start with a short introduction of concepts in category theory such as functors, natural transformations, representables and limits, followed by a proof of the Density Theorem. The theorem unifies all of these concepts and states that every presheaf is a colimit of representables, that is, representables are "dense" in the category of presheaves.

4- Adem Eren Uyanık (Supervisor: Aurelian Gheondea)

Classical Fourier Series

Abstract. In this report, we will first define trigonometric series and by means of them, Fourier series. Then we will seek for an answer to a major question: "When does the Fourier series of a function converge to that function?". Throughout this journey we will encounter some of the major results in analysis such as Fejer theorem, Riemann-Lebesque lemma, Parseval's identity, Tauberian type theorems and Dirichlet-Jordan theorem.

5- Alper Akyüz (Supervisor: Aurelian Gheondea)

Theory of Distributions and Applications to Partial Differential Equations

Abstract: The first half of this summer project has goal of studying Laurent Schwartz's theory of distributions, and present some of the most commonly used theorems with rigorous proofs, and define tempered distributions and Fourier transform on distributions. The second half of the project has the goal of applying theory of distributions and Fourier Transform on tempered distributions to 2 of the most commonly encountered partial differential equations, Poisson and Wave equations to study their solutions, and present some noteworthy results on classical theory of Laplace and Wave Equation.

6- Bahar İdil Ongun (Supervisor: Gökhan Yıldırım)

The Partially Asymmetric Simple Exclusion Process(PASEP)

Abstract: The Partially Asymmetric Simple Exclusion Process(PASEP) is a model mostly used in statistical mechanics introduced in the late nineties to describe a system of interacting particles hopping left and right on a one-dimensional lattice with n sites. PASEP is a discrete-time Markov chain, and it has a unique stationary distribution. Computing the stationary distribution is an important problem in understanding the long time behaviour of the model. Derrida et al. proposed a matrix ansatz as a solution to this problem. Later in 2007, Cortell and Williams made a connection between PASEP stationary probabilities and the generating function of a combinatorial structure, called "Permutation Tableaux," with respect to three statistics. This approach led to a new solution for computing the stationary probabilities of PASEP. We study the PASEP model with the two approaches mentioned above for computing the stationary probabilities of the model.

7- Faruk Beygo (Supervisor: Gökhan Yıldırım)

A Connection between Partially Asymmetric Exclusion Process and Permutation Tableux

Abstract: This project aims to study the connection between the Partially Asymmetric Exclusion Process (PASEP) and some combinatorial structures (Permutation Tableaux-PT). The PASEP is a foundational model for interacting particles in statistical mechanics. Notably, these particles exhibit bidirectional motion, wherein discrete probabilities control transitions in both left and right directions. A Markov chain (PT chain) on the permutation tableaux was introduced by Corteel and Williams in 2012. The PT chain can project to the PASEP model and gives a novel method to compute the stationary distribution of the PASEP. Our primary focus lies in studying the proof of the stationary distribution of the PT chain and how it projects to the PASEP.

8- Alihan Serim (Supervisor: Naci Saldı)

On Discrete-Time Markov Decision Processes

Abstract. This project aims to develop the foundational properties of Discrete-Time Markov Decision Processes (MDPs) used widely in different-type control problems. In control theory, we mathematically model a dynamical system whose behavior can be affected or controlled by an appropriate selection of some of the system's, control, variables. We aim to find the proper control policy that either minimizes or maximizes the performance criterion. In this project, we are concerned with

discrete-time control system where the time parameter of the system changes in a discrete sense. First of all, we give the definitions of a Discrete-Time Markov Control Model and a control policy. Then, we form the theorems in cases, where the system is operated during a finite time interval (Finite-Horizon Problems) and an infinite time interval (Infinite-Horizon Problems). In doing so, a proof of the Dynamic Programming (DP) Theorem for Finite-Horizon and the Value Iteration Theorem for Infinite-Horizon is studied. After that, the theory is applied to some example problems.

9- Serkan Doruk Hazinedar (Supervisor: Yaghoub Heydarzade)

Planetary Motion in the Theory of General Relativity

Abstract: In this project, we study planetary motion within the framework of Einstein's General Theory of Relativity (GR) from a mathematical point of view by establishing a strong enough foundation in differential geometry. The study begins with introducing basic ideas like manifolds, tangent-cotangent spaces, vector and tensor fields, and progresses to the Lorentzian manifolds, covering connections, geodesics, and curvature. Then, Einstein's field equations connecting the space-time geometry to energy-momentum sources are introduced. In the next step, the Schwarzschild solution as the vacuum solution of the field equations is studied. This solution is an exact solution to Einstein's field equations and describes the gravitational field outside a spherically symmetric mass and hence is suitable for studying planetary dynamics. As an application of GR, using the Schwarzschild metric, the project addresses the solution to the perihelion precession problem in the context of Newtonian gravity.