**Bilkent University**
**Department of Mathematics**

# Problem Of The Month

August 2005

**Problem:** Prove that the equation

$$x_1^3 + x_2^3 + x_3^3 + x_4^3 + x_5^3 = n$$

has an integer solution for any integer $n$.

**Solution:** First of all, note that any number $n = 6k$ can be represented as a sum of four cubes:

$$(k+1)^3 + (-k)^3 + (-k)^3 + (k-1)^3 = 6k.$$

Now we note that

$$n = 6k + 1 = 6k + 1^3$$
$$n = 6k + 2 = 6(k-1) + 2^3$$
$$n = 6k + 3 = 6(k-4) + 3^3$$
$$n = 6k + 4 = 6(k+2) + (-2)^3$$
$$n = 6k + 5 = 6(k+1) + (-1)^3.$$

Therefore, our equation has an integer solution for all values of $n$.