Bilkent University
Department of Mathematics

Problem Of The Month

January 2026

Problem:

The set of side lengths of four rectangles consists of 8 integers. If the sum of areas of two of these rectangles is equal to the sum of areas of the remaining two of these rectangles what is the minimal possible total area of these four rectangles?

Solution:

Answer: The minimal total area $S = 62$.

Let us give an example for $S = 62$: $1 \cdot 7 + 4 \cdot 6 = 2 \cdot 8 + 3 \cdot 5 = 31$.

Suppose that the set of side lengths is $\{a, b, c, d, e, f, g, h\}$. If non of the elements is 1 then

$$S \geq ab + cd + ef + gh \geq 4\sqrt[4]{abcdefgh} \geq 48\sqrt[4]{35/2} > 96.$$

Let $h = 1$. Then $S \geq ab + cd + ef + g \cdot 1$, where $g \geq 8$ is a largest number. Then

$$S \geq g + 3\sqrt[3]{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7} > g + 51 \geq 60.$$

If $S = 60$ then the set of side lengths is $\{1, 2, 3, 4, 5, 6, 7, 8\}$. Then for some a, b, c we have $5a + cd = 30$ which is impossible: $5 \not| cd$. Done.