Bilkent University
Department of Mathematics

PROBLEM OF THE MONTH

Term: November 2024

At the beginning the board contains 77 vectors

$$(1, 0, 0, \ldots, 0), \ (0, 1, 0, \ldots, 0), \ \ldots, \ (0, 0, 0, \ldots, 1)$$

each having 77 components. At each step we choose two vectors $(a_1, a_2, \ldots, a_{77})$ and $(b_1, b_2, \ldots, b_{77})$ written on the board and write their sum $(a_1 + b_1, a_2 + b_2, \ldots, a_{77} + b_{77})$ to the board. Find the minimal number of steps which should be made to get all the vectors

$$(0, 1, 1, \ldots, 1), \ (1, 0, 1, \ldots, 1), \ \ldots, \ (1, 1, 1, \ldots, 0).$$

on the board.