Bilkent University
Department of Mathematics

Problem Of The Month

Term: May 2024

Find the maximal possible value of

$$\sum_{i,j=1,2,\ldots,n,\ i\neq j} \lceil x_i x_j \rceil - (n-1)\left(\sum_{i=1,2,\ldots,n} \lfloor x_i^2 \rfloor\right)$$

for all real numbers $x_1, x_2, \ldots, x_n$.

Note: For a real number x, $\lceil x \rceil$ is the smallest integer which is not less than x and $\lfloor x \rfloor$ is the largest integer not exceeding x: $\lceil 1.8 \rceil = 2$, $\lfloor 2.4 \rfloor = 2$ and $\lceil 3 \rceil = \lfloor 3 \rfloor = 3$.