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PROBLEM OF THE MONTH

Term: October 2023

Let \mathcal{S} be the set of all 2023 tuples $(x_1, x_2, \dots, x_{2023})$, where $x_i \in \{1, 2, \dots, 100\}$ for each $1 \leq i \leq 2023$. The subset $T \subset \mathcal{S}$ is said to be down-dense if for each $(x_1, x_2, \dots, x_{2023}) \in T$ any $(y_1, y_2, \dots, y_{2023})$ satisfying $y_i \leq x_i$ ($1 \leq i \leq 2023$) also belongs to T . The subset $T \subset \mathcal{S}$ is said to be up-dense if for each $(x_1, x_2, \dots, x_{2023}) \in T$ any $(y_1, y_2, \dots, y_{2023})$ satisfying $y_i \geq x_i$ ($1 \leq i \leq 2023$) also belongs to T . Find the minimal possible value of

$$f(A, B) = \frac{|A| \cdot |B|}{|A \cap B|},$$

where A and B are non-empty down-dense and up-dense subsets of \mathcal{S} , respectively.

Note: $|T|$ denotes the number of elements of a set T .