



Bilkent University
Department of Mathematics

PROBLEM OF THE MONTH

Term: May 2020

Let \mathbb{Z}^+ be the set of all positive integers. For each function $f: \mathbb{Z}^+ \rightarrow \mathbb{Z}^+$ and $\ell \in \mathbb{Z}^+$ let f_ℓ be a composite function $\underbrace{f \circ f \circ \cdots \circ f}_{\ell \text{ times}}$. Find all functions $f: \mathbb{Z}^+ \rightarrow \mathbb{Z}^+$ satisfying

$$(n-1)^{2020} < \prod_{\ell=1}^{2020} f_\ell(n) < n^{2020} + n^{2019}$$

for each $n \in \mathbb{Z}^+$.