Bilkent University
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Problem Of The Month

Term: January 2020

Let $m = p_1^{d_1} p_2^{d_2} \cdots p_k^{d_k}$ be the prime decomposition of a positive integer m and the "derivative" function $f(n)$ be defined by

$$f(m) = f(p_1^{d_1} p_2^{d_2} \cdots p_k^{d_k}) = d_1 d_2 \cdots d_k p_1^{d_1 - 1} p_2^{d_2 - 1} \cdots p_k^{d_k - 1}.$$

For a given positive integer L, the L "derivative" sequence is the sequence $\{a_n\}, n = 1, 2, \ldots$ defined by $a_1 = L$ and $a_{n+1} = f(a_n)$, $n > 1$.

We say that a sequence $\{a_n\}$ is not N repeating if $i \neq j$, $a_i = a_j$ implies that $min(i, j) > N$.

Prove or disprove that for each positive N there is a L "derivative" sequence which is not N repeating.