Bilkent University
Department of Mathematics

Problem Of The Month

Febuary 2015

Problem:

Is there a set of 2015 consecutive positive integers containing exactly 15 prime numbers?

Solution: The answer is *yes*.

For each positive integer n let $f(n)$ be the number of prime numbers among $n, n+1, \ldots, n+2014$. We will show that $f(k) = 15$ for some positive integer number k. First of all we note that

- $f(1) \geq 15$ since $2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47$ are prime numbers.
- $f(2016!+2) = 0$ since for each $2 \leq l \leq 2016$ the number $2016!+l$ is not a prime number.

Now note that by the definition for each positive n the difference $f(n+1) - f(n)$ is equal to $0, -1$ or 1. In other words, while n increases by 1, $f(n)$ can change only by 1. Thus, when n changes from 1 to $2016! + 2$, $f(n)$ smoothly (at most by 1) changes from some number exceeding 15 to 0. Therefore, for some integer $1 < k < 2016! + 2$ we have $f(k) = 15$. Done.