Bilkent University
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Problem Of The Month

January 2011

Problem:

M is the set of squares of the first 20 natural numbers:

$M = \{1^2, 2^2, 3^2, 4^2, \ldots, 20^2\}$.

We say that n is a **good** number, if in any subset of M of size n there are two elements a and b such that $a + b$ is a prime number. Find the smallest **good** number.

Solution:

The answer: $n = 11$.

Let $K = \{1^2, 3^2, 5^2, \ldots, 17^2, 19^2\}$. Since the sum of any two elements of K is not prime, $n \geq 11$.

Now let us show that $n \leq 11$. We partition M into 10 subsets of order 2 such that the sum of two elements in any subset is prime:

$M = \{1^2, 4^2\} \cup \{2^2, 3^2\} \cup \{5^2, 8^2\} \cup \{6^2, 11^2\} \cup \{7^2, 10^2\} \cup \{9^2, 16^2\} \cup \{12^2, 13^2\} \cup \{14^2, 15^2\} \cup \{17^2, 18^2\} \cup \{19^2, 20^2\}$.

Any subset of M having 11 elements contains both elements of least one of these 10 subsets. Done.