Bilkent University
Department of Mathematics

Problem Of The Month

July-August 2010

Problem:

Are there 2010 points on the plane such that

i) any three of the points are non collinear
ii) the distance between any two points is irrational
iii) any triangle with vertices at given points has a rational area?

Solution:

Let $A_k = (k, k^2)$ for $k = 1, 2, \ldots, 2010$. Then

i) any three of the points are non collinear: all points lie on a parabola and the intersection of a parabola and a straight line contains at most 2 points

ii) the distance between any two points is irrational: $dist(A_m, A_n) = \sqrt{(m-n)^2 + (m^2-n^2)^2} = |m-n| \cdot \sqrt{1+(m+n)^2}$

iii) any triangle with vertices at given points has a rational area: for example, by Pick's theorem (the area of any polygon with vertices located on grid points is $a + b/2 - 1$, where a is the total number of interior points and b is the number of boundary points) the area any triangle is rational.