Bilkent University
Department of Mathematics

Problem Of The Month

November 2009

Problem:

Suppose that the set of all natural numbers N is partitioned into 3 pairwise disjoint infinite sets A, B and C: $A \cup B \cup C = N$. Prove that there are infinitely many triples $a \in A, b \in B$ and $c \in C$ such that a, b and c are sides of some triangle.

Solution:

Assume that there are only finitely many triples $a \in A, b \in B$ and $c \in C$ such that a, b and c are sides of some triangle. Since the sets A, B and C are infinite, there exist natural numbers $a_1 \in A, b_1 \in B$ and $c_1 \in C$ exceeding all these triangle sides and satisfying $1 < a_1 < b_1 < c_1$. Obviously, there is a triangle with sides a_1, c_1, c_1+1, as well as a triangle with sides $b_1, c_1, c_1 + 1$. Therefore, by assumption, $c_1 + 1 \in C$. By repeating this argument, we get that all natural numbers exceeding c_1 belong to C, and as a consequence the sets A and B are finite. A contradiction.