Bilkent University
Department of Mathematics

Problem Of The Month

December 2007

Problem:
Let $\{a_n\}$ be an increasing sequence of positive integer numbers. The term a_k of this sequence is said to be *good* if $a_k = t_l a_l + t_m a_m$ for some indices l and m and some positive integer numbers t_l and t_m. Prove that all but finite number of terms of this sequence are *good*.

Solution:

Suppose that a_k and $a_l; k > l$, are two terms of the sequence $\{a_n\}$ having the same remainder modulo a_1:

$$a_k = a_l + b \cdot a_1,\ b > 0,$$

meaning the term a_k is *good*. Therefore, no two different not *good* terms may have the same remainder modulo a_1. Thus, the total number of not *good* terms is at most a_1.