Bilkent University
Department of Mathematics

Problem Of The Month

May 2007

Problem: For all positive a, b, c satisfying $a + b + c = 1$, prove the following inequality:

$$\frac{1}{a(2-a)+bc} + \frac{1}{b(2-b)+ac} + \frac{1}{c(2-c)+ab} \geq \frac{9}{2}$$

Solution: By Arithmetik - Harmonic mean inequality

$$\frac{1}{a(2-a)+bc} + \frac{1}{b(2-b)+ac} + \frac{1}{c(2-c)+ab} \geq \frac{9}{2(a+b+c)-a^2-b^2-c^2+ab+bc+ac}$$

The denominator of the right hand side

$2(a+b+c) - a^2 - b^2 - c^2 + ab + bc + ac = 2 - (a^2 + b^2 + c^2 - (ab + bc + ac))$

$= 2 - \frac{1}{2}((a-b)^2 + (b-c)^2 + (a-c)^2) \leq 2.$

Done. The equality holds iff $a = b = c = \frac{1}{3}$.