Bilkent University
Department of Mathematics

Problem Of The Month

March 2006

Problem: Prove that $\sqrt{1+\sqrt{2+\sqrt{3+\cdots+\sqrt{2006}}}} < 2$.

Solution: We denote $\sqrt{1+\sqrt{2+\sqrt{3+\cdots+\sqrt{2006}}}}$ by A.

Note that $\sqrt{2005+\sqrt{2006}} < \sqrt{2005+46} < 46$.

Therefore, $A < \sqrt{1+\sqrt{2+\sqrt{3+\cdots+\sqrt{2004+46}}}}$.

By the same way

$A < \sqrt{1+\sqrt{2+\sqrt{3+\cdots+\sqrt{2003+46}}}}$.

By proceeding we get

$A < \sqrt{1+\sqrt{2+\sqrt{3+46}}} = 2$.